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Thermoelectric effects in superconducting nanostructures

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Abstract. We study thermoelectric effects in superconducting nanobridges and demonstrate that the magnitude of these effects can be comparable or even larger than that for a macroscopic circuit. It is shown that a large gradient of the electron temperature can be realistically created on a nanoscale and the masking effects due to spurious magnetic fields can be minimised in nanostructures. For these reasons, nanodevices can provide an interesting possibility to study the thermoelectric effect in superconductors.

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1 Introduction

The discrepancy between the theory and experiment concerning the thermoelectric phenomena is a long standing problem in physics of superconductors. The thermoelectric phenomena in the superconducting state were first discussed by Ginzburg [1] as early as 1944 (see also Ref. [2]). In the presence of a temperature gradient ∇T, there appears in a superconductor a normal current of the form given by

$$\boldsymbol{j}_T = -\eta \nabla T \tag{1}$$

where η is the corresponding transport coefficient. As was pointed out by Ginzburg [1], the normal current is offset by a supercurrent density $\boldsymbol{j}_s$ so that the total current in the bulk of a homogeneous isotropic superconductor should vanish

$$\boldsymbol{j}_T + \boldsymbol{j}_s = 0. \tag{2}$$

This makes impossible standard studies of the thermoelectric effect in a homogeneous isotropic superconductor. Ginzburg considered also simply-connected anisotropic or inhomogeneous superconductors where it is possible to observe thermoelectric phenomena by measuring the magnetic field generated by a temperature gradient.

Theory of the effect was further developed in 1970s [3] and the thermoelectric coefficient η in equation (2) was calculated using the Boltzmann equation approach [4]. One of the prediction was that $\eta(T)$ is a continuous function of temperature and at the critical point T_c it approaches its value of the normal metal $\eta(T_c)$; this is a robust feature independent of the mechanism of the quasiparticle scattering. It was noted in particular that the offset supercurrent is related to the phase difference of the order parameter of a simple-connected superconductor. The phase difference can be measured either in superconducting interferometer or in the loop formed by different superconductors where a magnetic flux is generated by a temperature difference. The theoretical development stimulated experimental study of the thermoelectric magnetic flux (the interferometry measurement has not yet been performed).

The first thermoelectric flux measurement by Zavaritsky [5] was in a reasonable agreement with the existing theory. However, further experiments [6,7] exhibited temperature-dependent magnetic fluxes some five order of magnitude larger than predicted by the theory [3]. Moreover, the experiment [6,7] gives a quite unexpected temperature dependence of the magnetic flux Φ near T_c. According to reference [7], it corresponds to the temperature dependence of the thermoelectric coefficient $\eta(T) \propto (T_c - T)^{-1/2}$, in a drastic contradiction to the theory of reference [3]. So far no explanation has been suggested for the unexpectedly huge effect and the mysterious temperature dependence of η.

A possibility to generate a large thermoelectric flux is discussed in [8] where it is related to the phonon drag effect near the interface of the two superconductors with different superconducting gaps. However, the predicted enhancement factor, the ratio of the Fermi and Debye energies, is not big enough to bridge the gap between the experiment [6,7] and the theory [3], to say nothing about the temperature dependence of the thermoelectric coefficient.

From the experimental point, the main difficulty is due to the fact that the thermoelectric effect is small, and one needs to single it out from various masking effects. The most obvious one is related to the temperature dependence

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of the magnetic field penetration length [9,10]. As a result, in the presence of a background magnetic field, the magnetic field within the superconductor is temperature dependent. This can mask the genuine thermoelectric effect which is due to the system being out of equilibrium.

The authors of references [6,7] study the thermoelectric effect in an ingenious device — a torus formed by a bimetallic superconducting loop. In this geometry, the thermoelectric magnetic flux is concentrated in the inner space of the torus while the measuring coil is winded around the torus. The contribution of a background magnetic field is minimal in the torus geometry but, in our opinion, the masking effects may be not excluded completely: Indeed, the measuring coil is outside the torus and thus is subjected to any external field penetrating the insulating gap between the coil and the torus. In an ideal geometry, the effect would be zero but due to asymmetry of a real sample, a spurious flux seems to be unavoidable. Since the area covered by this gap is essentially a macroscopic one, a background magnetic field can produce a significant magnetic flux through the gap which can be temperature-dependent, in particular, due to the temperature dependence of the penetration length as is discussed in references [9,10]. Thus one cannot be sure that the setup of references [6,7] excludes all the masking effects originating from the background magnetic field.

It is important to note that later on it was shown that the co-existence of a temperature gradient and a supercurrent leads to variation of the gauge invariant scalar potential ϕ; the latter describes the nonequilibrium charge related to an imbalance between the electron-like and hole-like excitations [11–14]. In contrast to the thermoelectric flux, the experimental studies of this effect were in agreement with the theory [14,15].

Note that there is a vast number of papers (see Refs. [16–27]) where thermoelectric effects in hybrid nanodevices including both normal and superconducting metals were studied. However we would like to emphasise that the measured quantity in these experiments was standard thermoelectric voltage, which vanishes for a superconducting branch. Thus thermoelectric effects in these studies originated from normal branches (possibly affected by superconductors through the proximity effects).

For almost three decades, there exist a challenging problem in the theory of nonequilibrium superconductivity: how one can reconcile the existing theory with the experimental data on the thermoelectric magnetic flux reported in references [6,7]?

The goal of the present paper is to discuss an entirely different, as compared to the former experiments, geometry of experiment (see Fig. 1): it includes a superconductor nanostructure as a part of a superconducting loop. The thermoelectric current is generated in the nanobridge, and the current is detected by the measurement of the magnetic flux in the loop. As discussed below, the masking effects due to redistribution of magnetic flux are small provided the transverse sizes of the wires of the loop are small compared to the London penetration depth. Due to development of experimental technique during recent years these experiments seem to be feasible. We believe that it would be interesting to compare the experimental data with the theory developed in the present paper. The comparison, particularly in the vicinity of T_c, may shed light on the origin and the very existence of a singularity of the thermoelectric transport coefficient observed in references [6,7].

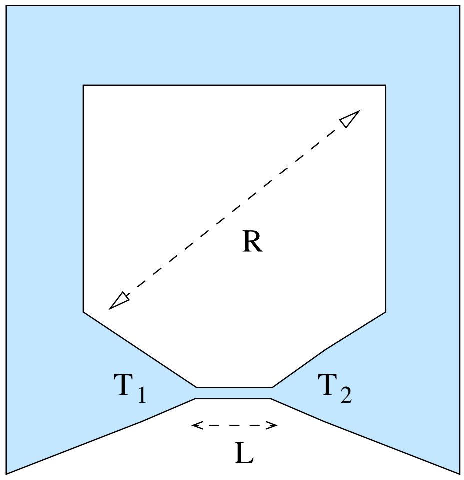

Fig. 1. Thermoelectric superconducting loop. The thermoelectric current is generated in a nanobridge the two banks of which are kept at different temperatures T_1 and T_2. The temperature difference is maintained by local heating of one of the banks. The bridge is shortcut by a superconducting wire (film) of size R.

Experimentally, an advantage of a properly designed nanostructure is that one is able to create a significant drop of the electronic temperature on a short distance (see Appendices A and B for details). Consequently, the temperature gradients are large so that the intrinsic thermoelectric current becomes larger and easier to observe. To realise such a favourable possibility, the design of the device must minimise the heat flow in the substrate the superconducting bridge is deposited on. Large temperature gradients can be achieved if the substrate is made of a material with a low thermal conductivity, for instance, a glass.

On the theoretical side, a re-analysis is needed because the earlier theory of the thermoelectric effect in superconductors considered bulk samples. Their sizes have been assumed to be much larger than the characteristic lengths such as the London penetration length, and the length at which the offset supercurrent is generated. When applied to bulk samples, there is no need to specify and go into detail of the mechanism by which the normal thermoelectric current is converted into the offset supercurrent. This approach is valid provided the sample size essentially exceeds the size of the region where the normal thermoelectric current $\boldsymbol{j}_n$ is converted into the offset supercurrent.

It is well-known from the microscopic theory [28] that the conversion occurs via generation of the nonequilibrium charge of the normal component, the charge imbalance, and subsequent relaxation of the latter. The charge imbalance relaxes due to the scattering events where the electron-like excitations are scattered into the hole-like branch of the excitation spectrum and vice verse. The microscopic mechanism of the branch-mixing, that is, the charge imbalance relaxation, is known to be the inelastic scattering, the impurity scattering for anisotropic superconductors, and the Andreev reflection provided inhomogeneity of the order parameter is present. When the bulk scattering dominates, the conversion takes place along the charge imbalance diffusion length L_b, $L_b = \sqrt{D\tau_b}$, D and τ_b being the diffusion constant and the charge imbalance relaxation time, respectively. For a nanostructure of the size comparable with the charge relaxation length, the standard theory of the thermoelectric phenomena (that assumes local compensation of the thermoelectric current) is not applicable. Indeed, in this case the normal thermoelectric current can be offset also by a normal diffusion current rather than by a supercurrent [28].

Additionally, there are important differences in electrodynamics of superconducting nanostructures. In particular, it is related to the so-called kinetic inductance $\mathcal{L}_k$

$$\mathcal{L}_k = \frac{L\lambda_L^2}{S}. \tag{3}$$

Here λ_L is the London penetration length, L is the wire length while S is the wire cross-section. In the case of a nanobridge, the inductance $\mathcal{L}_k$, which is proportional to S^{-1}, may be larger than the magnetic inductance of the thermoelectric loop for small enough values of S. In this case, the local compensation of the current in equation (2) turns out to be energetically unfavourable and the electrodynamical part of the theory requires a revision too.

In what follows we will develop a theory of thermoelectric effect in superconducting nanostructures. It will include the above kinetic and electrodynamical effects.

2 Thermoelectric current

To establish notation we first briefly overview electrodynamics of superconductors in the presence of the temperature gradient.

The total electric current density, $\boldsymbol{j} = \boldsymbol{j}_s + \boldsymbol{j}_n$, is a sum of the supercurrent $\boldsymbol{j}_s$, and normal $\boldsymbol{j}_n$, components. The supercurrent reads

$$\boldsymbol{j}_s = \frac{c^2}{4\pi e\lambda_L^2}\boldsymbol{p}_s \tag{4}$$

where $\boldsymbol{p}_s$ is the superfluity momentum,

$$\boldsymbol{p}_s = \frac{\hbar}{2}\nabla\chi - \frac{e}{c}\mathbf{A}, \tag{5}$$

χ and $\mathbf{A}$ being the phase of the order parameter and the vector potential, respectively.

The normal current density

$$\boldsymbol{j}_n = \boldsymbol{j}_T + \boldsymbol{j}_D, \tag{6}$$

is a sum of the thermoelectric current $\boldsymbol{j}_T$, and the diffusion component $\boldsymbol{j}_D$, related to the charge imbalance specified by the gauge invariant potential ϕ,

$$\phi = \frac{\hbar}{2e}\dot{\chi} + \varphi, \tag{7}$$

φ being the scalar potential.

In the vicinity of the critical temperature T_c the diffusion current $\boldsymbol{j}_D = -\sigma\nabla\phi$ is proportional to the normal state conductance σ, and the thermoelectric current $\boldsymbol{j}_T = -\eta\nabla T$ is controlled by the normal state thermoelectric coefficient η.

2.1 Diffusion limit

Consider two superconducting banks connected by a narrow wire of the length L and cross-section S (see Fig. 1). The transverse sizes of the wire are assumed to be much smaller than the London penetration length λ_L. In this case, the current is distributed homogeneously within the wire cross section, and the problem is one dimensional. Denote by x the coordinate along the wire and choose the origin in the middle of the wire. We analyse a diffusive wire and assume that the temperature varies linearly along the wire, its values at the banks being T_L and T_R. Note that the thermoelectric current is considered a constant equal to $-\eta\nabla T$ in the wire and zero in the banks. This assumption holds for 3D structures where both the temperature gradient and electric current density quickly decay within the contact. Naturally, we assume that the thickness of the wire is much smaller than the thicknesses of the banks.

The potentials $\boldsymbol{p}_s$ and ϕ are found from the continuity equation $\operatorname{div}\boldsymbol{j} = 0$, and the equation which describes transformation of the normal current into supercurrent that results in the following equation for ϕ in the wire (see, for instance, Ref. [4])

$$\nabla^2\phi - \frac{\phi}{L_b^2} = 0. \tag{8}$$

Here τ_b is the charge imbalance relaxation time while $L_b = \sqrt{D\tau_b}$ is the charge imbalance relaxation length [29]. If the banks are made of superconductors with different values of the gap equation (8) requires a boundary condition [30] to account for the Andreev reflection at the interface. The latter plays the role of a surface mechanism of the charge imbalance relaxation.

The boundary condition to equation (8) rather generally takes the form [30]

$$\frac{1}{\sigma}j_n\bigg|_{x=\pm L/2} = \pm\frac{1}{\mathcal{A}}\,\phi\bigg|_{x=\pm L/2} \tag{9}$$

where j_n is the x-component of the normal charge current density in equation (6), and $\mathcal{A}$ is an effective relaxation length controlled by the Andreev reflection at the wire-bank interface as well as the imbalance relaxation rate in the banks.

In the present paper, we are interested in the opposite limit of a short wire, $L \ll L_b$. The potential difference in this limit is

$$\Delta\phi = \frac{j_T}{\sigma}\frac{1}{\frac{1}{2\mathcal{A}}+\frac{1}{L}}. \quad (10)$$

For a short enough wire,

$$L < \mathcal{A}, \quad (11)$$

we obtain

$$\Delta\phi \sim \frac{\eta}{\sigma}\Delta T. \quad (12)$$

In this case of a short superconducting wire, the thermoelectric potential difference $\Delta\phi$ *is of the order of that in the normal state.* In a short wire, the supercurrent is homogeneous,

$$j_s = j_0 - j_T\frac{L}{2\mathcal{A}+L}, \quad (13)$$

where, as above, j_0 is the total electric current through the wire.

2.2 Ballistic bridge

When we studied the charge imbalance in the previous section, we have exploited the diffusive approximation. However, the largest values of j_T correspond to the largest values of the mean quasiparticle free path within the wire. So one expects the largest effect for a ballistic bridge. In this case one can estimate the quasiparticle thermoelectric current with the help of a procedure similar to the one used in reference [31]. Namely, one has in mind that the quasiparticle distribution function within the ballistic bridge is formed by the quasiparticles entering the bridge from the banks. One also notes that the distribution function is constant along the quasiparticle trajectory while the quasi-equilibrium distribution functions of the left and right banks correspond to different temperatures (T_L and T_R, respectively). Thus for the quasiparticle distribution function F_B, within the bridge one has

$$F_B = \theta\left(v_x\frac{\xi}{\varepsilon}\right)F(T_L) + \theta\left(-v_x\frac{\xi}{\varepsilon}\right)F(T_R). \quad (14)$$

Here v_x is the component of the electron velocity along the bridge direction, and $F(T_{L,R})$ stand for the equilibrium distribution function corresponding to the temperature $T_{L,R}$. We have taken into account that the (group) quasiparticle velocity differs from the "bare" electron velocity by a factor $\xi_{\boldsymbol{p}}/\varepsilon_{\boldsymbol{p}} = \xi_{\boldsymbol{p}}/\sqrt{\xi_{\boldsymbol{p}}^2+\Delta^2}$, $\xi_{\boldsymbol{p}}$ being the kinetic energy counted from the Fermi energy, and Δ being the superconductor energy gap. The normal thermoelectric current density for the distribution function given by equation (14) reads

$$j_T = 2e\int\frac{d^3p}{(2\pi\hbar)^3}v_xF_B.$$

As usual in the theory of thermoelectric phenomena, the contributions of electrons and holes to the current tend to cancel, and the net effect is sensitive to details of the band structure and to the energy dependence of the density of states, in particular. At temperatures T near T_c the order of magnitude of the thermoelectric current can be estimated as

$$j_T \sim ev_Fn\frac{(T_L^2-T_R^2)}{\varepsilon_F^2}, \quad (15)$$

where v_F and ε_F are the Fermi velocity and energy, respectively, and n is the electron density. For a rough estimate, assume that the temperature difference is comparable to T_c. In this case,

$$j_T \sim env_F\left(\frac{T_c}{\varepsilon_F}\right)^2. \quad (16)$$

Thus a presence of a temperature drop at the contact between two superconducting banks leads to formation of the thermoelectric current through the nanobridge. The order of magnitude of the current can be evaluated according to equation (16). The total thermoelectric current, I_T, is given by $I_T = j_TS$ where S is the area of the bridge cross-section.

3 Thermoelectric flux

As we have discussed above, we study a nanostructure (Fig. 1) that consists of a superconducting bridge with a thickness and a width smaller than the London penetration depth λ_L. The bridge joins two banks made of the same superconductor (with critical temperature T_{c1} and a thickness smaller than λ_L). By means of a point-like heating, the banks are kept at different temperatures. We assume that the bridge region carrying thermoelectric current I_T is short-circuited by superconducting branch with sizes larger than λ_L forming a loop of the linear size R. The behaviour of the system is different for the two limiting cases: a) $R \gg L_b$; b) $R \ll L_b$.

We start our analysis with the first one, that is the case when the charge imbalance relaxation length L_b is much shorter than the size of the system. Such a case can be realised in particular if the near-contact region at least for one of the banks is covered by a superconductor with a larger gap leading to effective imbalance relaxation due to Andreev reflections. If the circuit is simple-connected the thermoelectric current is compensated by the supercurrent created due to the Andreev reflection or bulk mechanisms of the charge imbalance relaxation. Thus the decay of the normal thermoelectric current is *locally* compensated by the supercurrent.

The situation becomes different if the circuit is not simple-connected, i.e., when another branch (made, for instance, of the material with a larger T_c) closes the loop. In this case, the net electric current through the bridge, being a sum of the normal thermoelectric and superconducting components, may be finite. Indeed, the charge current continuity is maintained by the supercurrent I_s through the branch closing the loop: I_s is actually the electric current circulating in the loop while $(I_s - I_T)$ is the superconducting component of the net current through the bridge. The circulating current I_s can be readily evaluated minimising the total energy W of the system. The latter is given by the following expression,

$$W = \frac{1}{2}(I_T - I_s)^2\mathcal{L}_k + \frac{1}{2}I_s^2\mathcal{L}. \quad (17)$$

The first term originates from the kinetic energy of superconducting electrons in the bridge, $\mathcal{L}_k$ being the well-known kinetic inductance given by equation (3). The second term in equation (17) is the energy of magnetic field created by the circulating current I_s, and $\mathcal{L}$ is the inductance of the loop, which is close to the geometrical inductance of the macroscopic branch. Minimising W with respect to I_s, one obtains

$$I_s = I_T\frac{\mathcal{L}_k}{\mathcal{L}_k + \mathcal{L}} \quad (18)$$

and, thus the thermoelectric magnetic flux is

$$\Phi_T = I_T\frac{\mathcal{L}_k\mathcal{L}}{\mathcal{L}_k + \mathcal{L}}. \quad (19)$$

The flux Φ_T is controlled by the smaller of the inductances in question.

Note that if $\mathcal{L}_k \ll \mathcal{L}$, Φ_T does not depend on $\mathcal{L}$ and is estimated as

$$\Phi_T = I_T\mathcal{L}_k \sim I_T\frac{L\lambda_L^2}{S}. \quad (20)$$

In the dirty limit, the penetration depth λ_L is related to its value in the bulk of a pure material as

$$\lambda_L^2 = \lambda_0^2(\xi_0/l_e)$$

where $\xi_0 \sim v_F/\Delta$ is the coherence length, and l_e is the electron elastic mean free path. As it can be seen, this result agrees with the physical picture considered in earlier papers [3,6] where it is assumed that the thermoelectric current is almost completely offset by the supercurrent.

However, the situation is qualitatively different if $\mathcal{L}_k \gg \mathcal{L}$, a condition which can be realistically met for a nanoscale bridge. Indeed, assuming $L \sim \sqrt{S}$ and $L \sim l_e \sim 10^{-6}$ cm, for $\lambda_0 \sim 10^{-5}$ cm, $\xi_0 \sim 10^{-4}$ cm, one obtains $\mathcal{L}_k \sim 10^{-2}$ cm. This means that the kinetic inductance $\mathcal{L}_k$ may be comparable to the magnetic geometric inductance $\mathcal{L} \sim R$ even for a relatively large, nearly macroscopic loop. In this case, the normal thermoelectric current generated by the bridge is non-locally short-circuited by the supercurrent through the macroscopic branch rather than being offset locally by the supercurrent. For this limit, one has from equation (19):

$$\Phi_T = I_T\mathcal{L}. \quad (21)$$

Despite the absence of the current cancellation within the bridge, the flux through the loop is $\mathcal{L}_k/\mathcal{L}$ times *smaller* than for the situation considered previously [3,6]. At the same time, the magnetic field within the structure practically coincides with its value for a normal metal structure. Correspondingly, *it can be much larger than for the thermoelectric effect in macroscopic circuits.* Indeed, assuming that the inductance $\mathcal{L}$ is of the order of the linear size of the circuit R, our estimates for the magnetic induction from equations (20), and (21) are

$$B_T \sim I_T\frac{\mathcal{L}_k}{\mathcal{L}^2} \quad , \quad \mathcal{L}_k \ll \mathcal{L} \quad (22)$$

and

$$B_T \sim \frac{I_T}{\mathcal{L}} \quad , \quad \mathcal{L}_k \gg \mathcal{L}. \quad (23)$$

Thus the "thermoelectric" magnetic field is the larger the smaller is $\mathcal{L}$ and is much larger for the regime $\mathcal{L}_k > \mathcal{L}$ than for a "macroscopic" regime considered earlier in references [3,6]. We believe that this factor significantly suppresses a possible role of masking effects.

In the limiting case (b), where the charge imbalance length L_b is much larger than the size of the system ($R < L_b$), the quasiparticle thermoelectric current is not converted into a supercurrent but short-circuited by the normal current through the closing branch (as it occurs in the normal metal thermoelectric circuits). The normal charge current in the loop generates a magnetic flux which in turn generates a circulating supercurrent I_c in the direction opposite to the normal current. In this case, the energy,

$$W = I_s^2\mathcal{L}_k/2 + (I_T - I_s)^2\mathcal{L}/2,$$

is built of the supercurrent kinetic energy $I_s^2\mathcal{L}_k/2$ and the magnetic energy $(I_T - I_s)^2\mathcal{L}/2$. Minimising W, one gets

$$I_s = I_T\frac{\mathcal{L}}{\mathcal{L} + \mathcal{L}_k},$$

and the total thermoelectric flux, $\Phi_T = (I_T - I_c)\mathcal{L}$, is again given by equation (19). Therefore, the thermoelectric flux Φ_T is completely controlled by the normal component provided $\mathcal{L}_k > \mathcal{L}$.

Let us estimate the largest possible values of Φ_T which can be realised for large $\mathcal{L}$. We have

$$\Phi_T \sim I_T\frac{L\lambda^2}{S} \quad (24)$$

where L and S are the bridge length and cross-section area respectively.

Correspondingly,

$$\Phi_T \sim env_F\left(\frac{T}{\mu}\right)^2\frac{L\lambda_0^2\xi_0}{l_e}. \quad (25)$$

In what follows we will assume that all the sizes of the bridge are of the same order while one should also put $l_e \sim L$. Assuming $T/\mu \sim 10^{-4}$ ($T \sim 1$ K), $\lambda^2 \sim 10^{-10}$ cm^2, $\xi_0 \sim 10^{-4}$ cm one has $\Phi_T \sim 10^{-3}\Phi_0$.

For smaller $\mathcal{L}$ the magnetic fluxes are smaller than the above estimate but the magnetic fields are higher.

4 Conclusion

In this paper we have analysed the thermoelectric effects in superconducting nanostructures. When the size of a thermoelectric circuit is smaller than the charge imbalance length, the very physical picture of the thermoelectric effects becomes different from that considered earlier for macroscopic systems. Indeed, the quasiparticle thermoelectric current rather than being offset locally by the supercurrent, is short-circuited nonlocally by the diffusion current in the branch closing the circuit. (Here one finds a certain similarity to the physical picture of the thermoelectric effect in thermoelectric loops made of two different normal metals.) The magnitude of thermoelectric effects in superconducting nanostructures may be comparable with that in systems of a macroscopic size. At the same time, the masking effects inherent for macroscopic superconductors can be eliminated so that nanoscale structures are promising for studying the thermoelectric effects in superconductors. In accordance with our estimates, the thermoelectric flux may be in the range $\Phi_T \sim 10^{-3}\Phi_0$, the value that should be readily measured by a SQUID-magnetometer.

For the thermoelectric effects to be detectable, the temperature drop must be large enough. However, even if a superconducting microstructure is deposited on a glass, it is still difficult to create a comparatively large temperature difference within a microcontact for the last is very small. One of the ways to circumvent this difficulty is to create *electron temperature* difference that may be much higher compared to the lattice temperature (see Appendix A). An important advantage of the nanobridge geometry is the possibility to have a large drop of an *electron* temperature T_e — similarly to a possibility to have a large voltage drop in a point contact [31]. In both cases, the relaxation of the nonequilibrium distribution function (related to the voltage or temperature drop) takes place deep in the bulk of the banks rather than within the nanobridge itself, provided the inelastic relaxation length is much larger than the size of the bridge. We have already mentioned that thermoelectric effects in hybrid normal metal-superconductor nanostructures related to the electron temperature drops were experimentally studied in a number of papers [16–27]. However in the latter studies the electron heating was produced by Joule effect due to external current. In our case any external current can produce a magnetic field which can imitate the thermoelectric flux. Thus we believe that to prevent such a masking effect one can thermally excite the electron system of the one of the banks with the help of a tunnel junction. In this case each of the tunnelling electrons can create $eV/2T_e$ electron-hole pairs, V being the tunnelling bias (see Appendix B for details). As a result, for a large enough bias one can obtain large values of T_e resulting from weak tunnel currents which do not interfere with the thermoelectric flux measurement.

Another interesting way of creating a comparatively large temperature and easily controllable temperature difference within a microstructure is to deposit in its vicinity a bimetallic stripe of two normal metals with the electric current through the stripe. Due to the Peltier effect, the heat will be released within one of the contacts between the metals and absorbed within the the other one, thus creating a temperature difference. Again, we expect to treat this situation in detail elsewhere.

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Appendix A: Temperature distribution

The purpose of this section is to discuss the conditions when one can ascribe different temperatures to electrons in two banks connected by a short bridge.

In recent years it has been demonstrated that the electronic temperature of a metal film may substantially differ from the lattice temperature of the dielectric substrate. For quasi-2D metallic nanostructures at low temperatures, there are two factors that are favourable for such a possibility. First, small electron-phonon collision rates prevent effective transfer of heat to the phonon system of the substrate. Second, the phonon heat conductivity of the substrate at small spatial scales turns out to be smaller than the electron heat conductivity within the films since the phonon mean free path is limited by the spatial inhomogeneity. Using the Wiedemann-Franz law, one estimates the electron heat current within the metal layer of a length L and cross-section S as

$$Q_{el} \sim \frac{\Delta T}{L} S D_e \tilde{n} \tag{26}$$

where ΔT is the temperature difference, D_e is electron diffusivity, and $\tilde{n} \sim (T/\varepsilon_F)n$ is the concentration of quasiparticles participating in the heat transfer (where n is the total electron concentration while ε_F is the Fermi energy). At the same time, the heat flux from the film to the substrate can be estimated as

$$Q_{\rm sub} \sim \frac{SL\tilde{n}\Delta T}{\tau_{\rm e\text{-}ph}} \tag{27}$$

where $\tau_{\rm e\text{-}ph}$ is electron-phonon relaxation time. From equations (26), and (27), one sees that $Q_{\rm el} > Q_{\rm sub}$ provided $L^2 < l_e v_F \tau_{\rm e\text{-}ph}$.

It is also instructive to compare the electronic heat flux Q_{el} with the heat flux Q_{ph} supported by phonons in the substrate and "shunting" the electron flux. One easily obtains that $Q_{el} > Q_{ph}$ provided

$$\frac{L}{d}\frac{s\min(w,l_{ph})}{v_F l_e}\left(\frac{T}{T_D}\right)^3\frac{\varepsilon_F}{T} < 1 \tag{28}$$

where T_D is the Debye temperature of the substrate, w is the width of the metal layer, d is the layer thickness, s is the sound velocity while l_e and l_{ph} are the mean free paths of electrons within the layer and phonons within the substrate, respectively. Since the electron heat conductivity dominates provided any of the aforementioned conditions holds, one concludes that at low temperatures the electron temperature is mainly controlled by electron heat conductivity of the metal structure.

Consider now a point ballistic bridge connecting two metal banks with different electronic temperatures. It follows from the above considerations that the temperature drop is concentrated mainly within the contact region. Indeed, for 3D geometry and a diffusive transport in the bulk, the temperature distribution in the banks near the bridge follows the same law as an electric potential distribution, that is the temperature drop is concentrated in the bridge. If the whole structure is made of a metal film of the same thickness and with the diffusive electron transport this statement holds only with a logarithmic accuracy because of the 2D character of electron diffusion. However if the thickness of the bridge region is much smaller than the thicknesses of the banks (that is if the configuration is a 3D-like one) the temperature drop is again completely restricted by the contact region. The same holds provided the electron transport within the contact and near-contact regions is ballistic. Indeed, it follows from the fact that under the Wiedemann-Franz law the temperature profile is similar to the electric potential profile while in 2D ballistic structures the potential drop is concentrated in the contact region.

It is expected that very large values of ΔT can be realised in the point contact geometry. Indeed, one can apply for the heat flux the same arguments as for electric current through the point contact [31], namely, that the relaxation processes for the electrons take place within the bulk of the sample at distances $(\sim D_e \tau_{ee})^{1/2}$. Thus enormous values of temperature gradient and heat flux density do not lead to destruction of the bridge.

Appendix B: Electron heating

Let us consider the important practical question concerning the generation of the temperature gradient across the bridge. We have assumed above that the excitations within the one of the banks are heated as compared to the excitations in another one. Since we deal with superconductors, it excludes the Joule heating. On the other hand, microwave heating implies relatively large areas. In our opinion, the best way is to heat electrons on one on the banks using a tunnel S-I-N junction. The junction is formed by a normal film of area S_2 put on the top of the superconducting bank (with a thin insulating layer). When the bias eV across the S-I-N junction is much larger than the superconductor energy gap, high-energy electrons tunnelling from N layer will relax mainly due to creation of electron-hole pairs within the superconducting layer. To have the electron temperature formed, one should have

$$S_2 > v_F l_e \tau_{ee},$$

τ_{ee} being the electron-electron scattering time.

Now let us compare the thermal current from the heated superconducting layer to the substrate and the thermal current through the point contact to the "cold" bank. Assuming that the thickness of the superconducting layer forming the tunnel junction and the layer forming the point contact are the same, one finds that the thermal current through the contact dominates provided

$$S_2 L/w < l_e v_F \tau_{\text{e-ph}} \tag{29}$$

where L and w are the point contact length and width, respectively. If $L \sim w$ one concludes, that this condition can hold since at low enough temperatures $\tau_{\text{e-ph}} > \tau_{ee}$. Correspondingly, in this case only a region with the area S_2 (under the tunnel junction) is heated with respect to the rest of the device, the heat leak being due to thermal current through the point contact. Certainly, one should also assume that the area of the superconducting layer in the "cold" bank is large enough to ensure efficient heat withdrawal to its substrate. In this case one easily obtains

$$\Delta T \sim IV \frac{L\varepsilon_F}{wdD_e nT} \tag{30}$$

where I is the current through the tunnel junction.

The main conclusion following from the considerations given above is that it is possible to have "point-like" electron heating restricted by the area $\sim v_F l_e \tau_{ee}$. Its linear dimensions for realistic estimates can be as small as 3 μm. Correspondingly, if the inductance loop has macroscopic size this local heating (and corresponding local variation of the penetration length) is not expected to affect the temperature-dependent (or rather V-dependent) flux through the loop.

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